October 12, 2004 Jacek Szmigielski

Math 238 Test # 1

No aids: no calculators, closed book. You are not permitted to consult with your fellow students in any way. Time: 80 minutes.

Part I, no partial credit

Question 1. (Two questions, each worth 1 pt) Consider the sequence defined by $a_n = \frac{\cos n}{n}$. (a) Is $\{a_n\}$ increasing, decreasing, or neither? (b) Determine whether $\{a_n\}$ converges or diverges, find its limit.

Question 2. (Three questions, each worth 1 pt) Consider the three infinite series below.

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5n}, \quad (ii) \sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$$

$$(iii) \sum_{n=1}^{\infty} \frac{5(-4)^{n+2}}{3^{2n+1}}$$

(a) Which of these series is (are) alternating?
(b) Which one of these series diverges, and why?
(c) One of these series converges absolutely. Which one? Compute its sum.

Question 3. (Four questions, each worth 1 pt) Find the radius of convergence and the interval of convergence of the following series.

(a) $\sum_{n=1}^{\infty} 5^n x^n$.
(b) $\sum_{n=1}^{\infty} \frac{x^n}{n^5}$
(c) $\sum_{n=1}^{\infty} n^n x^n$
(d) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{(n!)^2}$

Question 4. (Two questions, each worth 1 pt) The Maclaurin series for e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
(a) Find the Maclaurin series for $f(x) = x^2 e^x$.
(b) Compute $f^{(100)}(0)$.

Question 5. (2 pts) Using the power series representation of $\frac{1}{1+x}$, find a power series representation of $f(x) = x \ln(1+x)$, which holds for $|x| < 1$.

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Part II, partial credit possible if warranted

Question 6. (2 pts) Is it true that 0.999..., with 9 repeated infinitely many times, equals 1? Give a proof if this statement is true, or, if untrue, explain the reason why this is a false statement.

Question 7. (2 pts) Using the integral test determine whether $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$ converges or diverges?

Question 8. (4 pts) Determine if $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ is absolutely convergent, conditionally convergent, or divergent. In each case state a theorem (in its entirety) to support your claim.

Question 9. (3 pts) Find a power series representation for $f(x) = \arctan(x/3)$ and determine its radius of convergence.

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